How to Incorporate Mathematical Elements into Your Writing

Mike Pierce · Tended 24 March 2024 · Hosted at coloradomesa.edu/~mapierce2/writing

Writing in a mathematical context is just writing all the same. The style and content may be more technical, but the underlying rules have not changed. It's still crucial that you write with an *audience* in mind, and that you craft your prose with the purpose of *delivering an impression* to your reader. This holds true even if you are a student; if you're writing for an exam, then your audience is the grader, and your purpose is to convince them of your proficiency and understanding.

When you start to introduce mathematical elements into your writing, as you must inevitably do if you plan to use math in your work, you must learn how the underlying rules should be interpreted to accommodate them. One natural way to learn this is by reading many examples, through which you will "acclimate to the norms" of mathematical writing simply through exposure. However for some explicit instruction, here are three bits of mathematical writing advice to focus on.

Respect an expression's syntactic category

Mathematical symbols are stand-ins for words or short phrases, and as such belong to a syntactic category (i.e. a part-of-speech). For example, functions like f and variables like x and numbers like 3 are all nouns, whereas the equality symbol "$=$" is usually a verb. Mathematical expressions like "$3x-1$" are *expressions* in the grammatical sense, whereas an equation like "$f(x)=3x-1$," having a verb, is a full clause.

It's important to place mathematical expressions in writing in a way that's grammatically correct. When beginning, it can be helpful to describe every mathematical expression explicitly as the type of mathematical object it is. For example, write *"the function f,"* or *"the acute angle $\angle ACB$,"* or *"the vector $\langle 2,3,5\rangle$."* Or for a narrative example:

> The line consisting of all points (x,y) in rectangular coordinates such that $y=3x-1$ is inclined at an angle of $\arctan(3)$. Defining the function f by the formula $f(x)=3x-1$, the graph $y=f(x)$ will coincides with this line. The function f is invertible, with inverse defined as $f^{-1}(x)=\frac{1}{3}(x+1)$. The graph $y=f^{-1}(x)$ is notably also a line, this one inclined at an angle of $\arctan(1/3)$. But since this graph must be the reflection of the original line over the diagonal line where $y=x$, it must be that $\arctan(3)+\arctan(1/3)=\frac{\pi}{2}$.

If you're ever in doubt over whether your mathematical writing is grammatically correct, read it out loud. Since you likely speak more than you write, your ear is trained to hear awkward, incorrect language better than your eye is trained to recognize it in writing.

Treat large mathematical expressions as figures

Generally, sufficiently large mathematical expressions should be treated the same as figures (e.g. images, tables, charts) in your writing, and removed from the main flow of text. For an example, look over this writing sample, being sure to focus more on the *arrangement* and *flow* of the text over its content.

> For this investigation my point of departure is provided by the observation of *Euler* that the product
>
> $$\prod \frac{1}{1-\frac{1}{p^s}} = \Sigma \frac{1}{n^s},$$
>
> if one substitutes for p all prime numbers, and for n all whole numbers. The function of the complex variable s which is represented by these two expressions, wherever they converge, I denote by $\zeta(s)$. Both expressions converge only when the real part of s is greater than 1; at the same time an expression for the function can easily be found which always remains valid. On making use of the equation
>
> $$\int_0^\infty e^{-nx}x^{s-1}\,dx = \frac{\Pi(s-1)}{n^s}$$
>
> one first sees that
>
> $$\Pi(s-1)\zeta(s) = \int_0^\infty \frac{x^{s-1}\,dx}{e^x-1}.$$
>
> If one now considers the integral
>
> $$\int \frac{(-x)^{s-1}\,dx}{e^x-1}$$
>
> from $+\infty$ to $+\infty$ taken in a positive sense around a domain which includes the value 0 but no other point of discontinuity of the integrand in its interior, then this is easily seen to be equal to
>
> $$(e^{-\pi si}-e^{\pi si})\int_0^\infty \frac{x^{s-1}\,dx}{e^x-1},$$

From Wilkins' translation of Riemann's *On the Number of Prime Numbers less than a Given Quantity*

Small mathematical elements like $\zeta(s)$ and $+\infty$ fit comfortably inline, and can be easily read within the flowing narration of the body text, whereas larger expressions like

$$\prod(s-1)\zeta(s) = \int_0^\infty \frac{x^{s-1}\,\mathrm{d}x}{\mathrm{e}^x-1}$$

need more space on the page to breathe, and really can't be "read" but, like an image or chart or table, needs to be studied for a moment to be parsed. For an example closer to the undergraduate college curriculum, consider this fact from differential calculus:

The derivative of the function f defined by the formula $f(x) = x^{42}$ will have the formula $f'(x) = 42x^{41}$. We know this is true by the *power-rule*, but this could also be derived from first-principles, invoking the definition of the derivative:

$$42x^{41} = \lim_{h \to 0} \frac{(x+h)^{42} - x^{42}}{h}.$$

Again, notice how simple mathematical elements like the formulas for those functions can easily be read inline, whereas the rational expression in the limit is too bulky to fit in the preceding paragraph, and so should be given its own space.

Relegate irrelevant calculations to scratch paper

It's important to be mindful of the purpose of your writing. For much of the curricular mathematics you learned in elementary school, a computation *was* the purpose. Now as the mathematics you're encountering and writing about is becoming increasingly sophisticated, only sometimes will a computation be the purpose. If a given computation does not contribute to the purpose of your writing *and if you're confident your reader can infer the computation you've done and could reproduce it themselves*, then you may safely leave that computation out. That is, you may now abandon that unconditional demand from elementary school teachers that you "show your work."

> I have made this letter longer than usual because I did not have time to make it shorter.
>
> — Blaise Pascal, paraphrased from translation

Good writing should communicate an idea or deliver an impression to your reader as efficiently as possible. It should be succinct, to-the-point, and exactly as brief or extensive as the subject matter warrants. This is not only a courtesy to your reader, whose time you should value, but beyond efficiency it's a matter of maximizing the *effectiveness* of your writing; excessively verbose writing makes it more difficult for a reader to keep their focus on your main point. For example:

> **Task** • Prove that the three points $(1, -19)$ and $(-17, 123)$ and $(-71, 549)$ all lie on a common line.
>
> *Proof.* Note that the slope of the segment between the points $(1, -19)$ and $(-17, 123)$ is $-\frac{71}{9}$ and the slope of the segment between the points $(-17, 123)$ and $(-71, 549)$ is also $-\frac{71}{9}$. The defining feature of a line is its constant slope. I.e. the slope between any pair of points on a line is a constant, independent of the choice of points. Because these two slopes are the same then, the three points must lie on a common line with this slope.

To have written this proof, the author must have done the calculations

$$\frac{123 - (-19)}{-17 - (1)} = \frac{142}{-18} = -\frac{71}{9} \qquad \frac{549 - (123)}{-71 - (-17)} = \frac{426}{-54} = -\frac{71}{9}$$

but the author's purpose is *not* to teach his reader how to calculate the slope between two points. They're assuming their reader knows how to do this already. Instead the purpose is to demonstrate how to use facts about lines to prove those three points are co-linear. In fact, including those computations in the proof would take the reader's focus away from the argument that these slopes being equal is sufficient to conclude the points are co-linear.

Caution • Don't leave out computations that *are* crucial to your purpose! Doing so will signal to a reader that you're a crackpot. If you don't know if the computation is important, if you have any doubt at all, default to keeping it in your writing.

Granted, this advice is more natural to enact in the *editing* process than when writing an initial draft, but is still helpful to keep in mind while writing. Pragmatically, whenever you're writing about mathematics, always have a large collection of scratch paper handy; you inevitably have to do computations before you write about them, but you should ask yourself if a computation is crucial to you narrative and will be valued by your audeince before you transcribe it from your scratch paper to your draft.

Some Miscellaneous Tips

- The equality symbol "=" means something specific: be sure the expressions you write on either side of one are, indeed, equal, and don't use it in place of the correct conjunction. E.g. it's incorrect to write

  $$3x - 1 = 7 \quad = \quad 3x = 8.$$

  That middle "=" could be replaced by the word "so."

- The equality symbol does have two connotations to keep in mind. Sometimes it's *declarative*: writing "$3x - 1 = 7$" declares that $3x - 1$ is equal to 7. Sometimes though "=" is used to *assign* equality: writing "let $n = 42$" assigns the value of 42 to the variable n — they were not equal before that clause. Usually this distinction is clear from context, but sometimes you instead see the notation "$n \leftarrow 42$" to indicate assignment, especially in writing focused more on programming.

- The double-thick arrow "$\implies$" can be used in place of the English word "implies" or the phrase "which implies," which is useful for computation-heavy writing.

- A sentence should not start with a mathematical symbol. Instead the subject can be introduced with words. E.g. *"The function f is invertible."* instead of just *"f is invertible."*

Further Reading

A Guide to Writing Mathematics, by Kevin Lee
On Writing, by Terry Tao
Mathematical Writing, by Knuth, Larrabee, and Roberts